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**MATHEMATICAL MODELLING OF SALT PURIFICATION  
BY RECRYSTALLIZATION. A CROSS-CURRENT FLOW MODEL  
AND ITS COMPARISON WITH THE COUNTERCURRENT  
ARRANGEMENT**

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A model is derived for a multi-stage crystallization with cross-current flows of the solution and the crystals being purified. The purity of the product is compared with that achieved in the countercurrent arrangement. A suitable function has been set up which allows the cross-current and countercurrent flow models to be compared and reduces substantially the labour of computation for the countercurrent arrangement. Using the recrystallization of  $\text{KAl}(\text{SO}_4)_2 \cdot 12 \text{H}_2\text{O}$  as an example, it is shown that, when the cross-current and countercurrent processes are operated at the same output, the countercurrent arrangement is more advantageous because its solvent consumption is lower.

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In crystallization of a salt from a multicomponent system, the microcomponent is always entrained in the macrocomponent crystals. Although the entrainment may occur by different mechanisms, the distribution of the microcomponent at equilibrium can frequently be approximated by the so-called homogeneous distribution law

$$Y = k_H X, \quad (1)$$

where  $X$  and  $Y$  are the masses of the microcomponent per unit mass of the macrocomponent in the liquid (sol) and solid (c) phases, respectively. The macrocomponent will be refined by recrystallization provided  $k_H < 1$ . We assume that the equilibrium concentration of the macrocomponent in solution is independent of the microcomponent concentration and that

$$W_{\text{cq}} = W_{\text{eq}}(t), \quad (2)$$

where  $W_{\text{cq}}$  is the equilibrium mass of the macrocomponent per unit mass of solvent,

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and  $t$  is the temperature. In the present paper, a model of cross-current recrystallization is derived and compared with the countercurrent flow model discussed previously<sup>1,2</sup>.

## THEORETICAL

### Cross-Current Flow Model

We shall consider a steady-state cross-current recrystallization carried out in an  $n$ -stage cascade (Fig. 1). The mass streams of the macrocomponent,  $\dot{m}$ , leaving the  $k$ -th stage and their microcomponent concentrations  $X$  and  $Y$  will be denoted by the superscript  $k$ , i.e.  $\dot{m}^{(k)}$ ,  $X^{(k)}$ , and  $Y^{(k)}$ . The streams leaving each stage are considered to be in equilibrium, so that Eqs (1) and (2) are valid. It is assumed that the amount of the entering solution is the same for all stages. The entering solution contains  $\dot{m}_{\text{sol}}^{(0)}$  of the macrocomponent with a microcomponent concentration  $\tilde{X}^{(0)}$ , and the effluent solution after recrystallization has  $\dot{m}_{\text{sol}}^{(k)}$  of the macrocomponent and a microcomponent concentration  $X^{(k)}$ . The fresh solution is prepared by dissolving part of the product (completely free from the mother liquor) in pure solvent at the temperature of crystallization,  $t$ . The temperature is the same at all stages. Thus we have

$$\tilde{X}^{(0)} = Y^{(n)}. \quad (3)$$

The crystals (solid phase) at the entry and exit of the  $k$ -th stage contain  $\dot{m}_c^{(k-1)}$  and  $\dot{m}_c^{(k)}$  of the macrocomponent. The crystals carry with them the mother liquor which contains  $\dot{m}_m^{(k-1)}$  and  $\dot{m}_m^{(k)}$  of the macrocomponent, respectively. The relative concentrations of the microcomponent in the liquid streams of the  $k$ -th stage are  $X^{(k-1)}$  and  $X^{(k)}$  and those in the solid streams  $Y^{(k-1)}$  and  $Y^{(k)}$ .

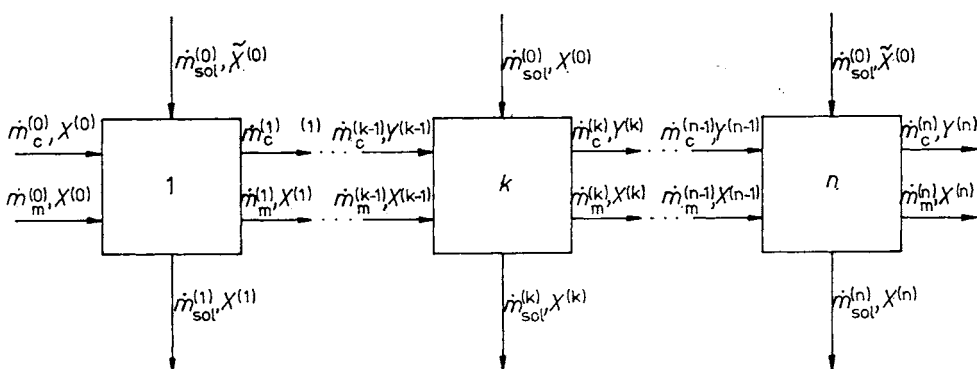


FIG. 1

Model of cross-current stagewise recrystallization

The crystals are dissolved and recrystallized at each stage. The relative mass fraction of the macrocomponent in the solution (based on the mass of the solvent) is  $W$  on crystal dissolution and  $W_{eq}$  on crystallization. The crystals generated at each stage are separated from the major portion of the mother liquor but are not washed and pass on together with the adhering solution to the next stage. If the equilibrium temperature is the same at all stages and the process is conducted so that the ratio  $\dot{m}_m^{(k)}/\dot{m}_c^{(k)}$  is constant, we can write

$$\begin{aligned}\dot{m}_m^{(o)} &= \dot{m}_m^{(k)} = \dot{m}_m = \text{constant} \\ \dot{m}_c^{(o)} &= \dot{m}_c^{(k)} = \dot{m}_c = \text{constant} \\ \dot{m}_{sol}^{(o)} &= \dot{m}_{sol}^{(k)} = \text{constant}.\end{aligned}\quad (4)$$

These assumptions also imply a constant moisture content of the crystals,  $U$  (expressed as the mass of solvent per unit mass of the solid phase).

The mother liquor,  $m_{sol}^{(k)}$ , is obtained by combining the mother liquors from all the stages. The mass flow rate of the macrocomponent is thus

$$\dot{m}_{sol} = \sum_{k=1}^n \dot{m}_{sol}^{(k)} = n\dot{m}_{sol}^{(o)}.\quad (5)$$

The average microcomponent level in the mother liquor is

$$\bar{X} = \frac{\sum_{k=1}^n \dot{m}_{sol}^{(k)} X^{(k)}}{\dot{m}_{sol}} = \frac{1}{n} \sum_{k=1}^n X^{(k)}.\quad (6)$$

The mass balance for the microcomponent on the  $k$ -th stage yields

$$\begin{aligned}\dot{m}_c Y^{(k-1)} + \dot{m}_m X^{(k-1)} + \dot{m}_{sol}^{(o)} \bar{X}^{(o)} &= \\ = \dot{m}_c Y^{(k)} + (\dot{m}_m + \dot{m}_{sol}^{(o)}) X^{(k)}, & [k = 1, 2, \dots, n].\end{aligned}\quad (7)$$

Let us introduce the following dimensionless parameters: the concentration

$$Z^{(k)} \equiv Y^{(k)}/Y^{(o)}, \quad [k = 0, 1, 2, \dots, n],\quad (8)$$

where  $Y^{(o)}$  is the relative mass fraction of the microcomponent in the crystals entering the first stage, and the recrystallization factor

$$K \equiv \frac{\dot{m}_{sol}^{(o)} \dot{m}_c}{k_H + \dot{m}_m / \dot{m}_c}.\quad (9a)$$

Equation (9a) can be rewritten in the alternative form

$$K = \frac{W_{\text{eq}}/(W - W_{\text{eq}}) - UW_{\text{eq}}/(1 - UW_{\text{eq}})}{k_{\text{H}} + UW_{\text{eq}}/(1 - UW_{\text{eq}})}, \quad (9b)$$

where, usually,  $1 \gg UW_{\text{eq}}$ . In the special (ideal) case where the crystals are completely free from the mother liquor, *i.e.* for  $U = 0$ , Eq. (9b) simplifies to

$$K^* = W_{\text{eq}}/[(W - W_{\text{eq}})k_{\text{H}}]. \quad (9c)$$

Using Eqs (1), (3), (8), and (9a), Eq. (7) can be written in the dimensionless form

$$k_{\text{H}}KZ^{(n)} - Z^{(k)}(1 + K) + Z^{(k-1)} = 0, \quad [k = 1, 2, \dots, n]. \quad (10)$$

The microcomponent content of the final product will be denoted as

$$Z_{\text{e}}^{(n)} \equiv Z^{(n)}. \quad (11)$$

Solving Eq. (10) for  $k = 1, 2, \dots, n$ , we find (see the Appendix)

$$Z_{\text{e}}^{(n)} = \frac{(1 + K)^{-n}}{1 - k_{\text{H}}[1 - (1 + K)^{-n}]}. \quad (12)$$

The compositions of the products from stages  $k = 1$  to  $(n - 1)$  can be obtained from Eq. (10) through the use of Eqs (11) and (12). Let us note that in the special (ideal) case where the fresh solution is saturated with a macrocomponent free from any microcomponent (as a first approximation, we may take  $1 \gg k_{\text{H}}[1 - (1 + K)^{-n}]$ ),  $\tilde{X}^{(0)} = 0$  and Eq. (12) reduces to

$$Z_{\text{e}}^{(n)} = (1 + K)^{-n}, \quad [\tilde{X} = 0, k_{\text{H}} < 1]. \quad (13)$$

The composition of the mother liquor can be determined from Eq. (6) where

$$X^{(k)} = Y^{(0)}Z^{(k)}/k_{\text{H}}. \quad (14)$$

The amount of crystals produced per unit volume of a crystallizer member (specific output) is

$$(\dot{m}_{\text{c}}/V)^{(k)} = \varrho_{\text{s}}(W - W_{\text{eq}})/\bar{\tau}, \quad (15)$$

where  $\varrho_{\text{s}}$  is the solvent density at the temperature  $t_{\text{eq}}$ ,  $\bar{\tau}$  is the average residence time

of the crystals, and  $V$  is the volume of solvent in each member. The crystal volume fraction,  $c$ , in cascade member is given by

$$c = \frac{\rho_s(W - W_{eq})/\rho_c}{1 + \rho_s(W - W_{eq})/\rho_c}, \quad (16)$$

where  $\rho_c$  is the macrocomponent density in the crystals. A part of the product is used to prepare the fresh solution. Therefore,

$$W - W_{eq} > nW_{eq} \quad (17)$$

or

$$W > (n + 1) W_{eq},$$

where  $n$  is the number of recrystallization stages. The yield per unit volume is then

$$(\dot{m}_c/V) = \rho_s[W - (n + 1) W_{eq}]/\bar{\tau}. \quad (18)$$

From Eqs (9b), (9c), and (17) it follows that

$$K < K^* < 1/nk_H. \quad (19)$$

Since the parameter  $k_H$  is determined independently (see Eq. 1), inequalities (17) and (19) are equivalent conditions defining the range of the validity of the model. Thus, if  $Z_e^{(n)}$  and  $n$  are predetermined in addition to  $k_H$ , we calculate  $K$  from Eq. (12). If the value of  $K$  does not satisfy condition (19), the process cannot be realized and new input parameters  $Z_e^{(n)}$  and  $n$  must be chosen.

### Countercurrent Flow Model

In previous papers<sup>1,2</sup> we developed a model of countercurrent crystallization. The model was based on the same assumptions as the present cross-current flow model except that the solution of the macrocomponent was fed into the last, *i.e.* the  $n$ -th stage, and the discarded solution was withdrawn from the first stage.

It has been shown that the mass balance equation for the microcomponent on the  $k$ -th member can be written in the form

$$Z^{(k-1)} - (1 + K) Z^{(k)} + KZ^{(k+1)} = 0, \quad [k = 1, 2, \dots, n]. \quad (20)$$

The recrystallization factor,  $K$ , is given by Eq. (9a) with  $\dot{m}_{sol}$  substituted for  $\dot{m}_{sol}^{(0)}$ .

The alternative expressions (9b) and (9c), however, remain unchanged. As shown previously<sup>1</sup>, it can be derived from Eq. (20) that

$$Z_e^{(0)} = 1 \quad (21a)$$

$$Z_e^{(1)} = (1 + K - \delta k_H K)^{-1} \quad (21b)$$

$$1/Z_e^{(n)} = (1 + K)/Z_e^{(n-1)} - K/Z_e^{(n-2)}, \quad n \geq 2. \quad (21c)$$

If the product from the  $n$ -th member is used to prepare the fresh solution, then  $\delta = 1$ , whereas if crystals free from any microcomponent are used for the same purpose, then  $\delta = 0$ . For predetermined values of the parameters  $k_H$ ,  $Z_e^{(n)}$ , and  $n$  in Eqs (21a,b,c), the numerical calculation of  $K$  for  $n \geq 3$  is facilitated by the limiting estimates<sup>3</sup>

$$K_{\min} < K < K_{\max},$$

where

$$K_{\min} = \sqrt[n]{\left(\frac{1 - \delta k_H Z_e^{(n)}}{Z_e^{(n)}(1 - \delta k_H)}\right)} \quad (22)$$

and

$$K_{\max} = \min \{ \sqrt[n]{1/Z_e^{(n)}}, 1/k_H \}.$$

Equations (15) and (16) are also valid for the countercurrent flow model. Condition (17), however, is transformed to

$$W - W_{eq} > W_{eq},$$

i.e.

$$W > 2W_{eq}. \quad (23)$$

The yield of the macrocomponent per unit volume is then given by

$$(\dot{m}_c/V) = \varrho_s(W - 2W_{eq}) \bar{v}. \quad (24)$$

Combining Eqs (9b), (9c), and (23), we find that

$$K < K^* < 1/k_H. \quad (25)$$

Inequalities (23) and (25) are alternative criteria that define the range of the validity of the countercurrent flow model. If, for example, the values of  $k_H$ ,  $Z_e^{(n)}$ , and  $n$  are

predetermined, then the parameter  $K$  calculated\* from Eqs (21a,b,c) must comply with condition (25).

## DISCUSSION

Comparison of the cross-current and countercurrent recrystallization processes can be made from several points of view. Here, we shall focus our attention on two cases of interest in chemical engineering:

- a) Assuming the same values of  $K$ ,  $k_H$ , and  $n$ , we shall compare the purity of the products.
- b) By considering a simple example, we shall compare the two models for the same yield of the crystals.

### *The Same Values of $K$ , $k_H$ , and $n$*

In the general case, the problem requires the three-dimensional analysis. As a first approximation, it may be handled two-dimensionally ( $K, n$ ) by assuming that the fresh solution is free from any microcomponent. For simplicity, we also assume that the mother liquor is completely removed from the crystals between the individual stages, so that  $U = 0$ . The ratio of the concentrations  $Z_e^{(n)}$  in the cross-current and countercurrent arrangements is then expressed by the function

$$J(K^*, n) = (1 + K^*)^n Z_e^{(n)}, \quad (26)$$

where  $Z_e^{(n)}$  is given by Eqs (21a,b,c) for  $\delta = 0$  and  $U = 0$ . The parameter  $K^*$  is defined by Eq. (9c). The function  $J(K^*, n)$ , whose shape is shown in Fig. 2, has the following properties:

$$J(K^*, 1) = 1 \quad (27)$$

and

$$J(K^*, n) = J(1/K^*, n). \quad (28)$$

The correctness of Eq. (27) can be shown from Eqs (21b) and (26). A proof of Eq. (28) is given in the Appendix. A graph of Eq. (28) is more convenient to use in determining  $Z_e^{(n)}(K^*, n)$  for the countercurrent arrangement than is a graph of Eqs (21) (presented previously<sup>1</sup> primarily for  $K^* < 1$ ). It should be pointed out that the function  $J(K, n)$  takes no account of the constraints on the validity of the model, *i.e.* of inequalities (19) and (25). These constraints must, however, be considered in checking the calculated values.

\* If even the  $K_{\min}$  calculated from Eq. (22) is found not to satisfy condition (24), then it is unnecessary to solve Eq. (21). In this case, a new choice of  $Z_e^{(n)}$  and  $n$  must be made.

*Example.* A four-stage recrystallization of  $\text{KAl}(\text{SO}_4)_2 \cdot 12 \text{H}_2\text{O}$  was carried out to remove  $\text{Na}^+$  ions. The crystals from each stage were completely freed of the mother liquor before entry to the next stage. The conditions of the process were chosen so that  $K^* = 3.1$ . Our task is to determine  $Z_e^{(4)}$ .

We shall find the answer first by using the nomogram in Fig. 2 and Eq. (26) and then by exact solution of the set of equations (21a,b,c).

For  $n = 4$  and  $K^* = 3.1$ , the nomogram gives  $J = 2.0$ . From Eq. (26) we obtain  $Z_e^{(4)} = 7.08 \cdot 10^{-3}$ . The exact solution of Eqs (21a,b,c) for  $\delta = 0$  is  $7.361 \cdot 10^{-3}$ . The results are in good agreement. The nomogram can be recommended for preliminary calculations at low values of  $U$  provided  $(1 + K) \gg k_H K$ .

### The Same Yield of Crystals

We consider a three-stage recrystallization of  $\text{KAl}(\text{SO}_4)_2 \cdot 12 \text{H}_2\text{O}$  in water, with an initial microcomponent concentration  $Y^{(0)} = 0.10$ . It is required that  $Y^{(3)} = 2 \cdot 10^{-5}$ , i.e.  $Z_e^{(3)} = 2 \cdot 10^{-4}$ . It has been established<sup>2</sup> that  $k_H = 0.082$  for rapid cooling (I) and  $k_H = 0.035$  for slow cooling (II). It is assumed that  $U = 0$ . According to published tables<sup>4</sup>,  $W_{\text{eq}} = W(20^\circ\text{C}) = 0.1127$ . The mean residence time of crystals in each member is 1 h. Our task is to determine the specific output per unit time and to compare the obtained value with that for the cross-current arrangement with  $n = 3$ . In doing so, we shall also compare the crystal volume fractions,  $c$ , for the two cases.

Solving the set of equations (21a,b,c) for  $\delta = 1$  and using Eq. (22), we find

$$K = 17.4 \quad [\text{variant I}]$$

$$K = 16.9 \quad [\text{variant II}].$$

Applying criterion (24), we find that the required purity  $Z_e^{(3)} = 0.2 \cdot 10^{-3}$  cannot be achieved by rapid cooling. The only feasible way to accomplish the task is variant II.

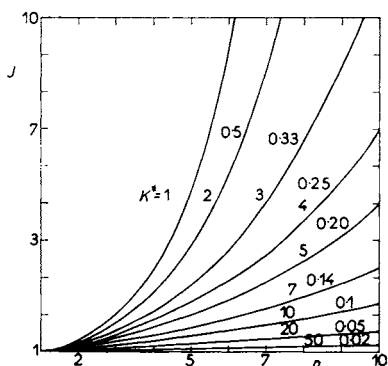


FIG. 2  
 $J(K^*, n)$  function defined by Eq. (16).



For variant II, Eq. (9c) gives

$$W/W_{\text{eq}} = 2.69 ,$$

*i.e.*

$$W = 0.303 .$$

According to solubility table<sup>4</sup>, the temperature of the solution in the crystallizer must be above 46°C. From Eq. (16) with  $\rho_s = 998.2 \text{ kg m}^{-3}$  and  $\rho_c = 1760 \text{ kg m}^{-3}$ , the crystal volume fraction at the equilibrium temperature is calculated as

$$c = 9.75 \cdot 10^{-2} .$$

The specific crystal output is found from Eq. (15) to be  $190 \text{ kg m}^{-3} \text{ h}^{-1}$ , and Eq. (24) gives the yield as  $77 \text{ kg m}^{-3} \text{ h}^{-1}$ . If the same yield is to be obtained in the cross-current arrangement, it follows from Eq. (18) for  $n = 3$  that

$$W/W_{\text{eq}} = 4.683 .$$

and for  $t_{\text{eq}} = 20^\circ\text{C}$

$$W = 0.528 .$$

This corresponds to a minimum temperature<sup>4</sup> of the solution in the crystallizer after the formation of the homogeneous phase of 58°C, *i.e.* by 12°C higher than that required in the countercurrent arrangement.

Thus, for  $n = 3$  condition (17) is satisfied and the process can be realized.

The parameter  $K^*$  for  $U = 0$  is obtained from Eq. (9c). We find that

$$K^* = 3.31 \quad [\text{variant I}]$$

$$K^* = 7.75 \quad [\text{variant II}] .$$

Using Eq. (12), we obtain

$$Z_e^{(3)} = 1.359 \cdot 10^{-2} \quad [\text{variant I}]$$

$$Z_e^{(3)} = 1.546 \cdot 10^{-3} \quad [\text{variant II}] .$$

Equation (16) gives the crystal volume fraction in both cases as

$$c = 0.186 .$$

By comparing the values of  $c$  and  $Z_e^{(3)}$  obtained from the two models, we arrive at the following conclusions. At the same yield as in the cross-current arrangement, the volumetric concentration,  $c$ , for the countercurrent process will be a factor of

1.9 lower. The crystal purity for slow cooling (variant II) in the countercurrent arrangement will be a factor of 7.7 higher. At the same time, the volume of the discarded solution will be a factor of 3 smaller. The temperature of the solution in the cross-current arrangement must be 12°C higher. The required purity of the product can be achieved by variant I, *i.e.* by rapid cooling, when applied to the cross-current arrangement, but not in the countercurrent process.

The above example shows that, at the same net output and the same number of recrystallization stages, the countercurrent arrangement is more advantageous.

### CONCLUSION

In most cases it is more advantageous to carry out the recrystallization in the countercurrent arrangement, which has an advantage over the cross-current process of a lower solvent consumption and hence lower energy requirements for handling the waste solutions. Under certain circumstances, however, a higher number of recrystallization stages may be required to attain the same degree of purity as in the cross-current arrangement. This may occur in practice if the recrystallization factor  $K < 1$ . From Eq. (9b) it follows that this case arises for  $W_{\text{eq}} \rightarrow 0$ . In such systems, the cross-current arrangement should be preferred.

### APPENDIX

*Proof of Eq. (12)*

Taking account of Eq. (11), Eq. (10) can be rewritten as

$$Z^{(k)} = (1 + K)^{-1} [Z^{(k-1)} + k_{\text{H}}KZ_e^{(n)}]. \quad (I-1)$$

According to Eq. (8),

$$Z^{(0)} = 1. \quad (I-2)$$

From the recurrence formula (I-1), it follows that

$$\begin{aligned} Z^{(1)} &= (1 + K)^{-1} + (1 + K)^{-1} k_{\text{H}}KZ_e^{(n)} \\ Z^{(2)} &= (1 + K)^{-2} + (1 + K)^{-1} k_{\text{H}}KZ_e^{(n)}[1 + (1 + K)^{-1}] \\ Z^{(3)} &= (1 + K)^{-3} + (1 + K)^{-1} k_{\text{H}}KZ_e^{(n)}[1 + K]^{-1} + (1 + K)^{-2}] \\ &\vdots \\ Z^{(n)} &= Z_e^{(n)} = (1 + K)^{-n} + (1 + K)^{-1} k_{\text{H}}KZ_e^{(n)} \sum_1^n (1 + K)^{-(n-1)}. \end{aligned} \quad (I-3)$$

The summation in Eq. (I-3) is a power series in  $(1 + K)^{-1}$ . The sum is given by

$$\sum_1^n (1 + K)^{-(n-1)} = \frac{1 - (1 + K)^{-n}}{1 - (1 - K)^{-1}}.$$

Therefore,

$$Z_e^{(n)} = (1 + K)^{-n} + k_H Z_e^{(n)} [1 - (1 + K)^{-n}],$$

and hence

$$Z_e^{(n)} = \frac{(1 + K)^{-n}}{1 - k_H [1 - (1 + K)^{-n}]}.$$

This completes the proof of Eq. (12).

*Proof of Eq. (19)*

The proof is by deduction. From Eq. (28) it follows that

$$J(K^*, 1) = J(1/K^*, 1) \quad (2-1)$$

since, identically,  $J(K^*, 1) = 1$  for an arbitrary  $K^*$ . Let us introduce an auxiliary function for which [ $K^* < 0$ ]:

$$K^*/(1 + K^*)^2 = (1/K^*)/(1 + 1/K^*)^2. \quad (2-2)$$

Let us multiply Eq. (21b) by  $(1 + K^*)^2$  and apply definition (26). Taking account of Eq. (2-2), we find that

$$J(K^*, 2) = J(1/K^*, 2). \quad (2-3)$$

Multiplying Eq. (21c) for  $n \geq 3$  by the expression  $(1 + K)^{-n}$ , applying again Eq. (26), and rearranging, we obtain

$$1/J(K^*, n) = 1/J(K^*, n-1) - K^*/(1 + K^*)^2 J(K^*, n-2). \quad (2-4)$$

From Eqs (2-4) and (2-2) it follows that

$$J(K^*, n) = J(1/K^*, n) \quad (2-5)$$

for a given  $n$  provided that it holds for  $(n-1)$  and  $(n-2)$ . On the basis of Eqs (2-1), (2-3), and (2-4), Eq. (2-5) holds for  $n = 3$ . If Eq. (2-5) holds for  $n = 2$  and 3 then, on the basis of (2-2) and (2-4), it must also hold for  $n = 4$ . By further deduction, if Eq. (2-5) holds for arbitrary  $(n-2)$  and  $(n-1)$ , then Eq. (2-4) implies that it must also hold for  $n$ . This completes the proof of Eq. (2-5) and hence Eq. (28).

## LIST OF SYMBOLS

$J(K^*, n)$	function defined by Eq. (16)
$K$	recrystallization factor defined by Eq. (9a)
$k_H$	homogeneous distribution coefficient
$m_c^{(k)}$	mass flow of macrocomponent in crystals leaving the $k$ -th stage
$m_m^{(k)}$	mass flow of macrocomponent in adhering mother liquor leaving the $k$ -th stage
$m_{sol}$	combined mass flows of macrocomponent in solutions leaving all stages
$m_{sol}^{(k)}$	mass flow of macrocomponent in solution leaving the $k$ -stage
$m_{sol}^{(o)}$	mass flow of macrocomponent in solution entering each stage
$n$	number of recrystallization stages
$t$	temperature of crystallization
$U$	relative mass fraction of solvent in crystal product
$V$	volume of solvent in the $k$ -th stage
$W$	mass of macrocomponent per unit mass of solvent after dissolution of all crystals in the $k$ -th stage and before recrystallization
$W_{eq}$	equilibrium mass of macrocomponent per unit mass of solvent at temperature $t$
$X^{(k)}$	mass of microcomponent in the liquid phase per unit mass of macrocomponent leaving the $k$ -th stage
$\tilde{X}^{(o)}$	mass of microcomponent in the liquid phase per unit mass of macrocomponent in fresh solution
$\dot{X}$	mass of microcomponent per unit mass of macrocomponent in combined effluent solutions from all stages
$Y^{(k)}$	mass of microcomponent in the solid phase per unit mass of macrocomponent in the $k$ -th stage
$Z^{(k)}$	relative concentration of microcomponent in crystals, defined by Eq. (8)
$Z_c^{(n)}$	relative concentration of microcomponent in crystal product from $n$ -stage recrystallization
$\tau$	time

## Superscripts and Overlays

(0)	input value
(k)	$k$ -th stage
(n)	$n$ -th stage
*	ideal state
—	mean value
~	input value

## Subscripts

c	macrocomponent in crystals
e	cascade output
eq	equilibrium
m	macrocomponent in solution adhering to crystals
s	macrocomponent in solution
sol	solvent

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